

**Physics Olympiad «Phystech»,
February 2023**

Problem Set 10-03

Ordinary fractions and radicals are allowed in your answers in all the problems of the set.



1. A projectile flies vertically and explodes at the highest point of the trajectory into many fragments flying in all possible directions with equal velocities in modulus. In $t_1 = 0.4$ s after the explosion all the fragments are in flight, one of the fragments moves horizontally, its momentum is $P_1 = 30$ kg·m/s. The mass of the projectile is $M = 10$ kg.

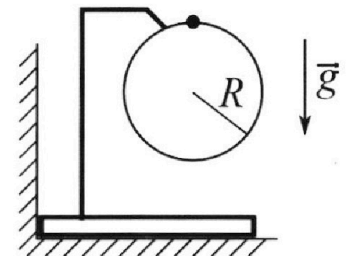
1) Find the module P_2 of the total momentum \vec{P}_2 of all the other fragments at this time point. Acceleration due to gravity is $g = 10$ m/s².

2) Find the angle α between the vectors \vec{P}_2 and \vec{g} at this time point. In the answer specify the value of the trigonometric function of the angle α : $\sin \alpha$ or $\tan \alpha$.

The maximum distance from the point of explosion to the ground point of impact equals $d = 80$ m.

3) Find the duration T of the flight of such fragments. Air resistance can be ignored.

2. The bar is installed close to the vertical wall (see Fig.). The ring of radius $R = 1$ m is fixed on the bar (the ring is in the vertical plane). A ball is put on the ring. The masses of the bar and the ball are the same. The ring and the holder are lightweight. There is no friction. From the upper point of the ring the ball slides with a negligible initial velocity.



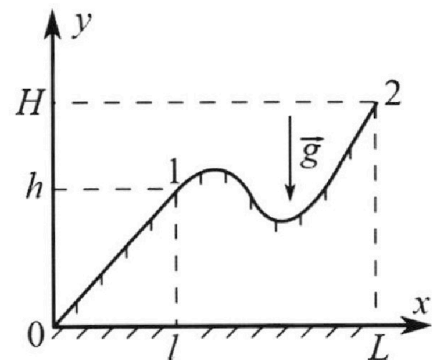
1) Find the acceleration \vec{a} of the ball at the moment when the force with which the bar acts on the vertical wall turns to zero. In the answer specify the module and direction of the vector \vec{a} .

2) Find the vertical displacement h of the ball to this point of time.

3) Find the maximum speed V of the bar movement.

Acceleration due to gravity is $g = 10$ m/s². In the process of movement the bar does not break away from the smooth horizontal surface.

3. A schoolboy pulls a sled on a hill moving in the straight line. Mass of the sled is $m = 5$ kg. The profile of the hill in the vertical plane is shown in the drawing for the task. In order to pull the sled slowly from point 0 to point 1, applying force along the flat surface of the hill, it is necessary to perform work $A_1 = 300$ J. At point 1 the schoolboy releases the sled. The vertical coordinate of the starting point is $h = 4.6$ m, the initial velocity of the sled is zero. The kinetic friction coefficient is the same on the entire surface of the hill. Acceleration due to gravity is $g = 10$ m/s².



1) Find the speed V of the sled at the base of the hill at point 0.

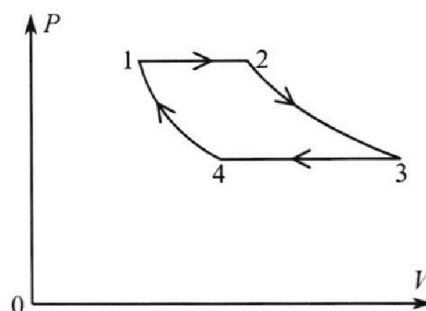
2) Find the work A_2 that should be done to move the sled slowly from point 1 to point 2? The vertical coordinate of the point 2 is $H = 10$ m, $L = 4l$. At each elementary displacement the vector of the force that the schoolboy applies to the sled and the vector of the sled displacement lie on the same straight line. All displacement occur in the same vertical plane.

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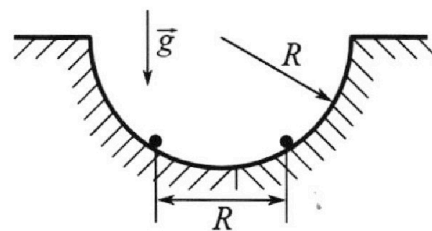
Ordinary fractions and radicals are allowed in your answers in all the problems of the set.

4. In the 1-2-3-4-1 cycle of the heat engine, there are two isobars and two isotherms (see Fig.). The working substance is a monatomic ideal gas. In the process of isobaric expansion up to double the volume, the gas performs the work A . The gas performs the same work A during isothermal expansion.



- 1) Find the amount Q of heat added to the gas in processes 1-2-3.
- 2) Find the amount Q_{34} of heat removed from the gas in the process of isobaric compression ($Q_{34} > 0$).
- 3) Find the efficiency e of the cycle.

5. A hemispherical hole of radius R is made in a smooth horizontal surface. Two charged balls are held in the hole at the same horizontal level (see Fig.). The mass of each ball is m , the distance between the balls is R . The balls are simultaneously released and they come off the hole at the edges. There is no friction. Counted from the edge of the hole the maximum height to which each ball rises in flight is equal to R .



- 1) Find the speed V of each ball in the beginning of the flight.
- 2) Find the charge Q of each ball.
- 3) Find the maximum speed U with which the distance between the balls grows after beginning the flight.

Acceleration due to gravity is g . The collisions of the balls with the horizontal surface are absolutely elastic. The proportionality coefficient in Coulomb's law is k .



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3) ~~By applying energy conservation~~
~~between points~~

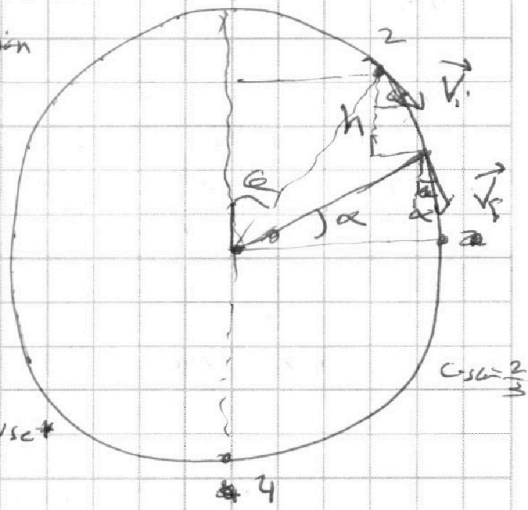
First by applying conservation

of momentum between times

when the force of the vertical

wall is zero (that is important because
there would be no forces on the
horizontal axis):

$\Delta P_x = 0$ between 2 and 4
 $P_i = P_f \Rightarrow m V_i \cos \alpha = m V_f \sin \alpha + m V \Rightarrow V_i \cos \alpha = V_f \sin \alpha + V$
 We have $V_i = \sqrt{\frac{2gR}{3}}$ from ~~the~~ ~~part~~ ~~part~~



and from conservation of energy:

$\Delta E = 0$
 $\Rightarrow \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2 + \frac{1}{2} m V^2 = mgh = mgR \cos \alpha - mgR \sin \alpha$
 put from (1) in (2)

$\frac{(V_i \cos \alpha - V)^2}{\sin^2 \alpha} - V_i^2 + V^2 = 2gR(\cos \alpha - \sin \alpha)$

$V^2 = 2gR(\cos \alpha - \sin \alpha) + V_i^2 - \frac{(V_i \cos \alpha - V)^2}{\sin^2 \alpha}$

V is max when $\frac{dV}{d\alpha} = 0 \Rightarrow -2gR \cos \alpha + \frac{2}{\sin^2 \alpha} (V_i \cos \alpha - V)^2$

$\Rightarrow 2gR \cos \alpha + \frac{2gR \cos \alpha (V_i \cos \alpha - V)^2}{\sin^2 \alpha}$

$\Rightarrow V^2 = 2gR(1 - \sin^2 \alpha) - \frac{2gR}{2g \sin^2 \alpha} - \frac{V^2}{\sin^2 \alpha} + \frac{4V}{3 \sin^2 \alpha} \sqrt{\frac{2gR}{3}}$

So $V = \frac{2 + \sqrt{16 - 9 \sin^2 \alpha}}{2(1 + \frac{1}{\sin^2 \alpha})} \sqrt{\frac{2gR}{3}} + \sqrt{\frac{16 - 9 \sin^2 \alpha}{9 \cdot 3 \sin^2 \alpha} - 4(1 + \frac{1}{\sin^2 \alpha})(2gR(\sin \alpha - 1) + \frac{4}{2g \cdot 3 \sin^2 \alpha})}$

So when $\frac{dV}{d\alpha} = 0$ $V = V_{max}$



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problem 2:

1) when no forces acting on the wall we have the normal force on the ball must be zero.

So:

$$\sum \vec{F} = m\vec{a}$$

by projecting: $mg \cos \alpha = N + F_c$

$$mg \cos \alpha = N + \frac{mv^2}{R}$$

$$N = 0 \Rightarrow mg \cos \alpha = \frac{mv^2}{R} \Rightarrow v^2 = Rg \cos \alpha$$

by applying conservation of energy:

$$\Delta E = 0 \Rightarrow \frac{1}{2}mv^2 = mgR(1 - \cos \alpha) \Rightarrow v^2 = 2gR(1 - \cos \alpha)$$

by solving (1) and (2):

$$Rg \cos \alpha = 2gR(1 - \cos \alpha) \Rightarrow \cos \alpha = \frac{2}{3}$$

the acceleration is perpendicular to the F_c so:

$$\sum \vec{F} = m\vec{a} \Rightarrow mg \sin \alpha = ma$$

$$a = g \sin \alpha = g \sqrt{1 - \cos^2 \alpha} = 10 \times \frac{\sqrt{5}}{3} \text{ m/s}^2$$

2) from the diagram: $h = R - R \cos \alpha$

$$h = R(1 - \cos \alpha) = 1 \times (1 - \frac{2}{3})$$

$$h = \frac{1}{3} \text{ m}$$

3)

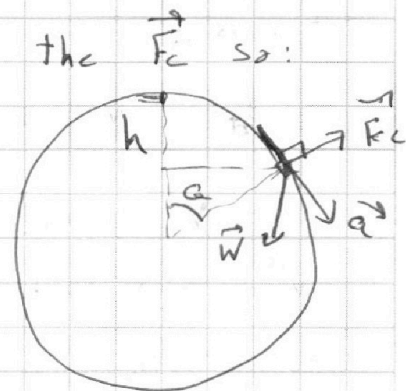
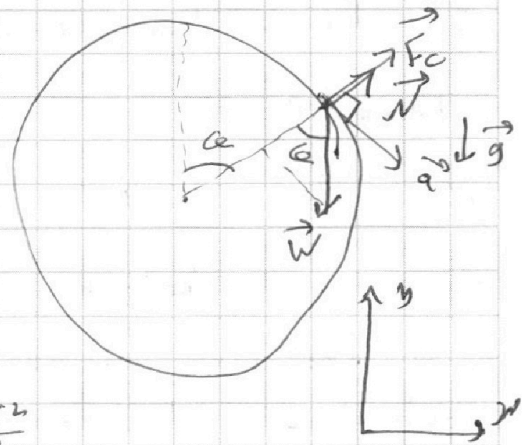
By conservation of momentum on the x axis:

$$p_{\text{ball}} = p_{\text{bar}} \text{ since } m_{\text{ball}} = m_{\text{bar}}$$

So we need to know the maximum

speed of ball on the x-axis, it is clearly that this point is the lowest point on ring so: $\Delta E = 0$

$$\Rightarrow \frac{1}{2}mv_{\text{max}}^2 = mg \cdot 2R \Rightarrow v_{\text{max}} = 2 \text{ m/s}$$





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Problem 3:

1) From conservation of energy we have:

$$\Delta E = -W_f + A_1$$

where W_f is the work of friction

$$\textcircled{1} mgh = -W_f + A_1$$

when we release it:

$$\Delta E = -W_f$$

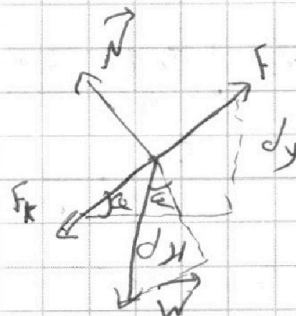
$\textcircled{2} \frac{1}{2}mv^2 - mgh = -W_f$ from $\textcircled{1}$ and $\textcircled{2}$ we have:

$$\frac{1}{2}mv^2 - mgh = mgh - A_1 \Rightarrow v = \sqrt{\frac{2}{m}(2mgh - A_1)}$$

$$v = \sqrt{(4 \times 10 \times 4,6 - \frac{300 \times 2}{5})} = \sqrt{184 - 120} = 8 \text{ m/s}$$

$$\textcircled{2} \sum \vec{F} = m\vec{a}$$

$$\Rightarrow N = mg \cos \alpha \text{ and } F_f = \mu N$$



$$dW = -F_f ds - mg \sin \alpha ds$$

$$dW = -\mu mg \cos \alpha ds - mg \sin \alpha ds$$

and we have $dx = ds \cos \alpha$ and $dy = ds \sin \alpha$

$$\text{so } dW = -\mu mg dx - mg dy$$

$$\textcircled{2} |W| = \int_{h}^L \mu mg dx + \int_h^H mg dy = 3\mu mg l + mg(H-h)$$

we have $A_1 = mgh + \mu mg l$ so $3\mu mg l = A_1 - mgh$

$$\text{by put } \textcircled{1} \text{ into } \textcircled{2}: |W| = 3(A_1 - mgh) + mg(H-h)$$

$$|W| = \frac{1}{2} A_2 = 3A_1 - 4mgh + mgH$$

$$A_2 = 900 - 920 + 500 = 480 \text{ J}$$



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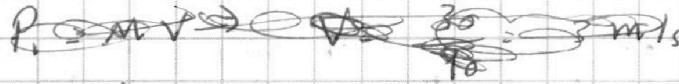
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Problem [1]:

1)



$$P_{zy} = M V_y \text{ and we have } V_y = \frac{1}{2} g t_1^2$$

because in ~~the~~ the highest point and because the projectile don't have momentum in the x -axis
So:

$$P_{zy} = P_z = M \frac{1}{2} g t_1^2 = \frac{10^1}{2} \times 10 \times (4 \times 10^{-1})^2$$

$$|P_z| = 2 \text{ kg m/s}$$

2) the projectile flies vertically so it doesn't have momentum in any axes but just in the axis ~~of~~ of vertical so $\vec{P}_z \parallel \vec{g}$ (parallel):

$$\text{so } \boxed{\sin \alpha = 0, \alpha = 0 \text{ rad}}$$

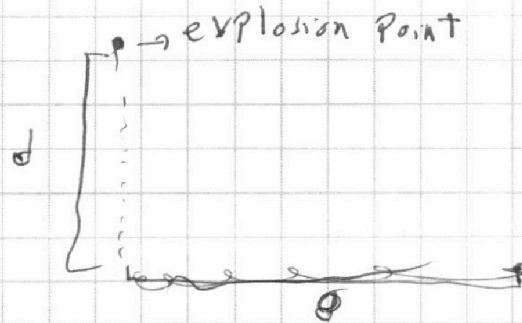
3)

$$d = \frac{1}{2} g T^2$$

$$T = \sqrt{\frac{2d}{g}}$$

$$T = \sqrt{\frac{2 \times 80}{10}}$$

$$\boxed{T = 4 \text{ sec}}$$





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3) we have: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ from: $P_1 V_1 = nRT_1$
and $T_3 = T_2$ and $T_1 = T_4$ because isothermal process
and $V_2 = 2V_1$ $P_2 V_2 = nRT_2$

so $T_2 = 2T_1$ Also we have $W_{23} = \int P dV = \int_{V_2}^{V_3} \frac{nRT}{V} dV$

$$\textcircled{1} W_{23} = nRT \ln \frac{V_3}{V_2} = 2P_1 V_1 \ln \frac{V_3}{V_2} = A$$

$$\text{and } \textcircled{2} W_{12} = P_1 (V_2 - V_1) = P_1 V_1 = A$$

$$\text{From } \textcircled{1} \text{ and } \textcircled{2}: \frac{V_3}{V_2} = e^{1/2} \Rightarrow V_3 = 2V_1 \sqrt{e}$$

$$\text{and in the same way we have } V_4 = \frac{V_3}{\sqrt{2}} = V_1 \sqrt{e}$$

$$\text{so } |Q_{14}| = P_1 V_1 \ln \frac{V_4}{V_1} = A \ln \sqrt{e} = \frac{A}{2}$$

$$\text{and } e = \frac{Q_{\text{eng}}}{Q_{\text{in}}} = \frac{|Q_{\text{in}}| - |Q_{\text{out}}|}{|Q_{\text{in}}|} = 1 - \frac{|Q_{\text{out}}|}{|Q_{\text{in}}|} = 1 - \frac{A + \frac{5A}{2}}{\frac{7A}{2}}$$

$$e = 1 - \frac{\frac{6A}{2}}{\frac{7A}{2}} = 1 - \frac{6}{7} = \frac{1}{7}$$

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Problem [4]:

1) Since it is a monatomic ideal gas $C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$

and in processes from (1) to (2):

we have:

$$\Delta E_{int} = Q - A$$

$$nC_V \Delta T = nC_P \Delta T - A$$

$$\Delta T = \frac{A}{n(C_P - C_V)} = \frac{A}{nR}$$

$$Q_{12} = nC_P \Delta T = n \frac{5}{2}R \frac{A}{nR} = \frac{5}{2}A$$

in isothermal process $\Delta E_{int} = 0$ so $Q_{23} = A$

$$\text{So } Q = Q_{23} + Q_{12} = A + \frac{5}{2}A = \frac{7}{2}A$$

2) We have $|dT_{34}| = |dT_{21}|$ because in isothermal process there is no change in temperature so $T_2 = T_3$
add $T_4 = T_1$ so $|dT_{34}| = |dT_{21}|$

$$\text{So } Q_{34} = nC_P \Delta T_{12} = |Q_{12}| = \frac{5}{2}A$$

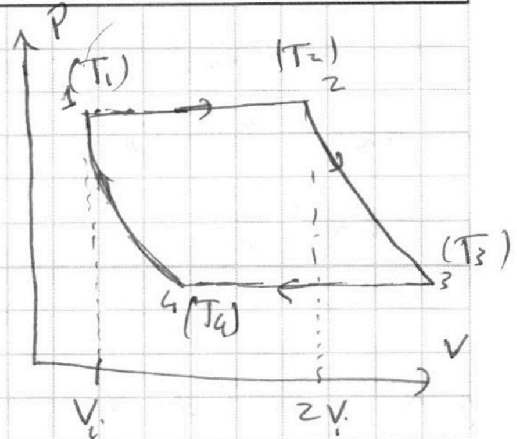
3) ~~We have $\Delta S = 0$ in any cycle so:~~

~~$$Q_{12} + Q_{23} - Q_{34} - Q_{41} = 0 \Rightarrow Q_{41} = \frac{7}{2}A + \frac{5}{2}A + A$$~~

~~$$Q = \frac{W_{eng}}{Q_h} = \frac{Q_{ht} - Q_{ct}}{Q_h} = 1 - \frac{Q_c}{Q_h} = \frac{A + \frac{5}{2}A}{\frac{7}{2}A + \frac{5}{2}A + A}$$~~

~~since it is a cycle so $\Delta E_{int} = 0$~~

~~$$W_{total} = Q_{total} = 0$$~~





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Problem 5:

1) by conservation of energy from the edge to the maximum height:

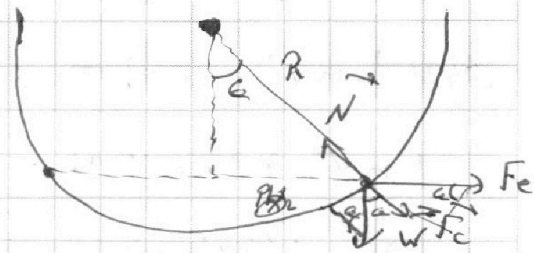
$$\Delta E = 0 \Rightarrow \frac{1}{2} m v^2 = m g R \Rightarrow \boxed{V = \sqrt{2gR}}$$

2) we have $dK = dW_e + dW$

$$dW = m g \sin \alpha R d\alpha$$

$$dW_e = \frac{k_c q^2 \cos \alpha R d\alpha}{d^2}$$

$$d = 2R \sin \alpha$$



$$dW_e = \frac{k_c q^2 R}{4R^2 R} \frac{\cos \alpha d\alpha}{\sin^2 \alpha}$$

$$W_e = \int \frac{k_c q^2}{4R} \frac{\cos \alpha d\alpha}{\sin^2 \alpha}$$

$$U = \sin^2 \alpha$$

$$dU = 2 \sin \alpha \cos \alpha d\alpha$$

$$W_e = \frac{k_c q^2}{8R} \int \frac{dU}{U^{3/2}} = \frac{k_c q^2}{8R} \left[-2 \frac{1}{\sin \alpha} \right]_{\alpha_1}^{\pi}$$

$$W_e = \frac{k_c q^2}{4R \sin \alpha_1}$$

We have $R \sin \alpha_1 = \frac{R}{2} \Rightarrow \sin \alpha_1 = \frac{1}{2}$
 $\boxed{\alpha_1 = 30^\circ}$

$$W_e = \frac{k_c q^2}{2R} \quad \text{and} \quad dW = W = m g R \cos 30 = \frac{m g R \sqrt{3}}{2}$$

$$\frac{1}{2} m v^2 = W_e - W \Rightarrow \frac{1}{2} m 2gR = -m g R \frac{\sqrt{3}}{2} + \frac{k_c q^2}{2R}$$

$$q = \sqrt{\frac{2 m g R^2 (1 + \sqrt{3})}{k_c}}$$

3) $\Delta E = 0 \Rightarrow 2 \left(\frac{1}{2} m v^2 \right) = \frac{k_c q^2}{2R}$

$$U_{\max} = q \sqrt{\frac{k_c}{2Rm}}$$

maximum speed when the distance between the balls become ∞ so $V_E = 0$ as $\boxed{V_E \propto \frac{1}{d}}$



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$$V_y = \frac{V_2 M_2 + V_3 M_3}{M_1 + M_2 + M_3}$$

$$P_y = (M_1 + M_2 + M_3) \vec{V}_{y,cm}$$

$$P_y = M_{total} \vec{V}_{y,cm}$$

$$P_{y,cm} = 0 \quad P_x = mV \quad W = nRT \ln \frac{V_f}{V_i}$$

$$W = \int P dV \quad T_f = \frac{T_i}{2}$$

$$P_1 V_1 = P_2 V_2 \quad P_1 V_1 = nRT_i \quad P_2 V_2 = nRT_f$$

$$\sum M_i \vec{v}_i = 0$$

$$J = V_2 t \quad \frac{V_3}{V_2} = e^2 \quad Q = -W$$

$$H = \frac{1}{2} g T^2 \quad \frac{2V_1}{V_2} e^2 \alpha W \quad Q = W \quad \frac{1}{L} = \frac{T_i}{T_f}$$

$$T_f = 2T_i$$

$$g \sqrt{1 - \frac{4}{3}}$$

$$g \cdot \frac{5}{3}$$

$$T_2 / T_1 = T_3 - T_4$$

$$\frac{V_3}{V_4}$$

$$mg \cos \alpha = \frac{mv^2}{R}$$

$$\frac{1}{2} mv^2 = mgR(1 - \cos \alpha)$$

$$v^2 = 2gR(1 - \cos \alpha)$$

$$2g(1 - \cos \alpha) = g \cos \alpha$$

$$2 - 2 \cos \alpha = \cos \alpha$$

$$3 \cos \alpha = 2$$

$$A = n P \cdot V_i$$

$$A = A h$$

$$A = P \cdot V_i$$

$$A = \frac{P_i \cdot 2V_i}{\sqrt{3}} (V_3 - V_4)$$

$$A = 2A - 2A \frac{V_4}{\sqrt{3}}$$

$$T_1 = T_4$$

$$\frac{P_1 V_1}{P_4 V_4} = P_1 V_1$$

$$\frac{V_3}{V_2} = e^2$$

$$n c v \alpha T = n c v \alpha T_0 + A$$

$$A = n R T \ln \frac{V_f}{V_i}$$

$$A = P_i V_i \ln \frac{V_f}{V_i}$$

$$W = \int P dV$$

$$\Delta E = Q - W$$

$$Q = \dots + A + A'$$

$$-A + A' \quad A = P_f (V_3 - V_4)$$

$$P_i V_i = P_f V_f$$

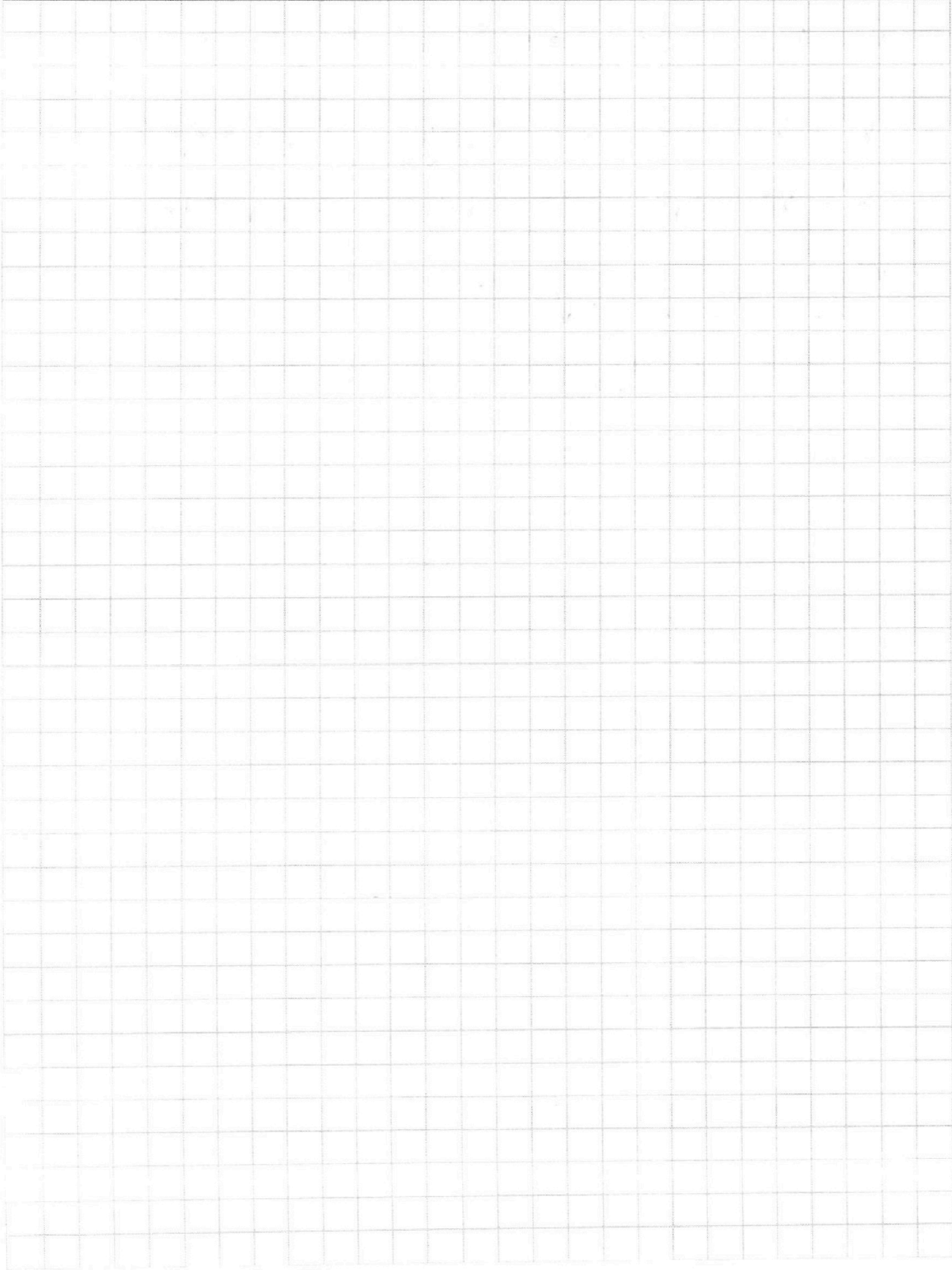


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$$V^2 = \frac{4gR}{3} - 2gR \sin \alpha + \frac{2}{3}gR - \frac{V_1^2 \cos^2 \alpha}{\sin^2 \alpha} - \frac{V^2}{\sin^2 \alpha} + \frac{2V V_1 \cos \alpha}{\sin^2 \alpha}$$

$$V^2 = 2gR - 2gR \sin \alpha - \frac{2gR}{3} \frac{4}{\sin^2 \alpha} - \frac{V^2}{\sin^2 \alpha} + \frac{2V \sqrt{\frac{2gR}{3}} \cos \alpha}{\sin^2 \alpha}$$

$$V^2 + \frac{V^2}{\sin^2 \alpha} - \frac{4V \sqrt{2gR}}{3} \frac{1}{\sin^2 \alpha} + 2gR \sin \alpha - 2gR + \frac{8gR}{27 \sin^2 \alpha} = 0$$

V^2

$$V = \frac{\frac{4}{3 \sin^2 \alpha} \sqrt{2gR} \pm \sqrt{\dots}}{2 \left(1 + \frac{1}{\sin^2 \alpha} \right)}$$



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Problem 2:

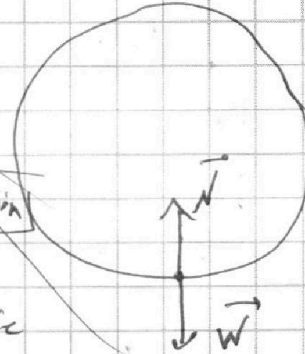
- 1) When the force with which the bar acts on wall is \neq zero it must be that no horizontal forces acting on the ball so that is when the ball is in its lowest point on the ring and we say that no horizontal forces acting on the ball so the $a_x = 0 \text{ m/s}^2$ and a_y also zero because of the normal force.

$$\vec{\Sigma F} = m\vec{a} \Rightarrow \Sigma F_y = ma_y$$

$$N = mg \text{ so } a_y = 0 \text{ and } a_x = 0$$

$$\text{So } \vec{a} = \vec{0} \text{ m/s}^2$$

No Friction



- 2) on its lowest point we have clearly $h = 2R = 2 \text{ m}$

3)